Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

0093871881

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 Investigation and Modelling (Extended) **October/November 2021**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 4) and part **B** (Questions 5 to 7).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

DC (PQ/CGW) 215051/4

Answer **both** parts **A** and **B**.

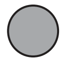
A INVESTIGATION (QUESTIONS 1 to 4)

CONNECTING DOTS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at the number of ways of connecting dots using straight lines.

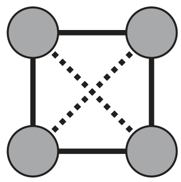
This diagram shows 1 dot.
There is 1 row and 1 column.
This is a 1 by 1 diagram.
There are no connections to other dots.



This diagram shows 4 dots.
There are 2 rows and 2 columns.
This is a 2 by 2 diagram.

There are 6 ways to join 2 dots.
These are:

- 2 vertical connectors (solid lines)
- 2 horizontal connectors (solid lines)
- 1 up diagonal connector (dashed line)
- 1 down diagonal connector (dashed line).

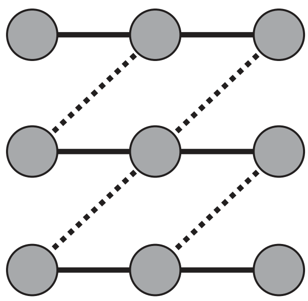


1 **(a)** This is a 3 by 3 diagram.

The diagram shows:

- 6 horizontal connectors
- 4 up diagonal connectors.

Each connector joins 2 dots.



Write down the number of vertical connectors and the number of down diagonal connectors that join 2 dots.

Vertical ....

Down diagonal .... [2]

(b) Complete the table for the numbers of connectors that join 2 dots.
Use **part (a)** and any patterns you notice.

You may use the square dotty paper on page 12 for diagrams.

Numbers of connectors that join 2 dots

Size of diagram (n by n)		Horizontal	Vertical	Up diagonal	Down diagonal	Total
	1 by 1	0	0	0	0	0
	2 by 2	2	2	1	1	6
	3 by 3	6		4		
	4 by 4				9	
	5 by 5	20				
	6 by 6					110

[3]

(c) In an n by n diagram there are n rows and n columns.

(i) Find an expression, in terms of n, for the number of up diagonal connectors that join 2 dots on an n by n diagram.

.... [2]

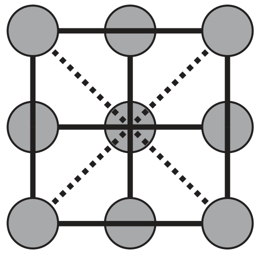
(ii) Find an expression, in terms of n, for the number of horizontal connectors that join 2 dots on an n by n diagram.

.... [3]

(iii) Use your answers in **part (i)** and **part (ii)** to find an expression for the total number of connectors that join 2 dots.
Do not simplify your expression.

.... [1]

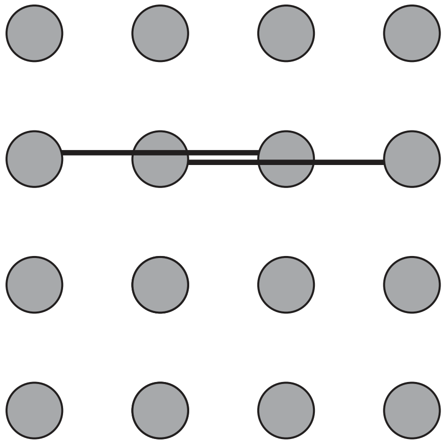
2 This is a 3 by 3 diagram.



There are 8 ways to join **3 dots** together.
These are:

- 3 vertical connectors
- 3 horizontal connectors
- 1 up diagonal connector
- 1 down diagonal connector.

(a) This is a 4 by 4 diagram.



Find the number of horizontal, vertical, up diagonal and down diagonal connectors that join 3 dots.
Two horizontal connectors have been drawn for you.

Horizontal ....

Vertical ....

Up diagonal ....

Down diagonal .... [2]

(b) Complete the table for the numbers of connectors that join 3 dots.
Use your answers to **part (a)** and any patterns you notice.

You may use the square dotty paper on page 12 for diagrams.

Numbers of connectors that join 3 dots

Size of diagram (n by n)		Horizontal	Vertical	Up diagonal	Down diagonal	Total
	2 by 2	0	0	0	0	0
	3 by 3	3	3	1	1	8
	4 by 4					
	5 by 5	15				
	6 by 6					80

[2]

(c) **(i)** This is an expression for the number of up diagonal connectors that join 3 dots on an n by n diagram.

$$(n-2)^2$$

Work out the number of up diagonal connectors that join 3 dots on a 20 by 20 diagram.

.... [1]

(ii) This is an expression for the number of horizontal connectors that join 3 dots on an n by n diagram.

$$n^2 + an$$

Find the value of a and write down the expression.

.... [3]

(d) Find an expression, in terms of n, for the total number of connectors that join 3 dots on an n by n diagram.
Do not simplify your expression.

.... [1]

3 **(a)** Complete the table for the numbers of connectors that join 4 dots.

Numbers of connectors that join 4 dots

Size of diagram (n by n)	Horizontal	Vertical	Up diagonal	Down diagonal	Total
3 by 3	0	0	0	0	0
4 by 4					10
5 by 5	10				
6 by 6					54
n by n					

[4]

(b) Find an expression, in terms of n, for the total number of connectors that join 4 dots on an n by n diagram.
Do not simplify your answer.

.... [1]

4 **(a)** This is an expression for the total number of connectors that join m dots on an n by n diagram.

$$2n(n-k)+2(n-k)^2$$

What is the relationship between k and m?

.... [1]

(b) **(i)** Use **part (a)** to show that when $n = 5$ and $m = 2$ there is a total of 72 connectors.

[1]

(ii) Find all the possible values for n and m that give a total of 72 connectors.

.... [3]

B MODELLING (QUESTIONS 5 to 7)

BREEDING DEER (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at modelling the population of a herd of deer.

In this herd there are equal numbers of males and females.
Each adult female deer gives birth to a male and a female fawn (baby deer) every year.

At the start of the first year a farmer has 20 adult deer.
There are 10 males and 10 females.
At the end of each year all the deer are adult.

5 **(a)** **(i)** At the end of the first year the farmer sends 20% of the 20 deer to other farmers.
He sends an equal number of male and female deer.

Show that at the start of the second year he has 36 deer in his herd.

[2]

(ii) P_n = the number of deer in the herd at the start of year n.
P_{n+1} = the number of deer in the herd at the start of year $n+1$.

At the end of each year the farmer sends 20% of P_n to other farmers.
He always sends an equal number of male and female deer.

Show that $P_{n+1} = 1.8P_n$.

[2]

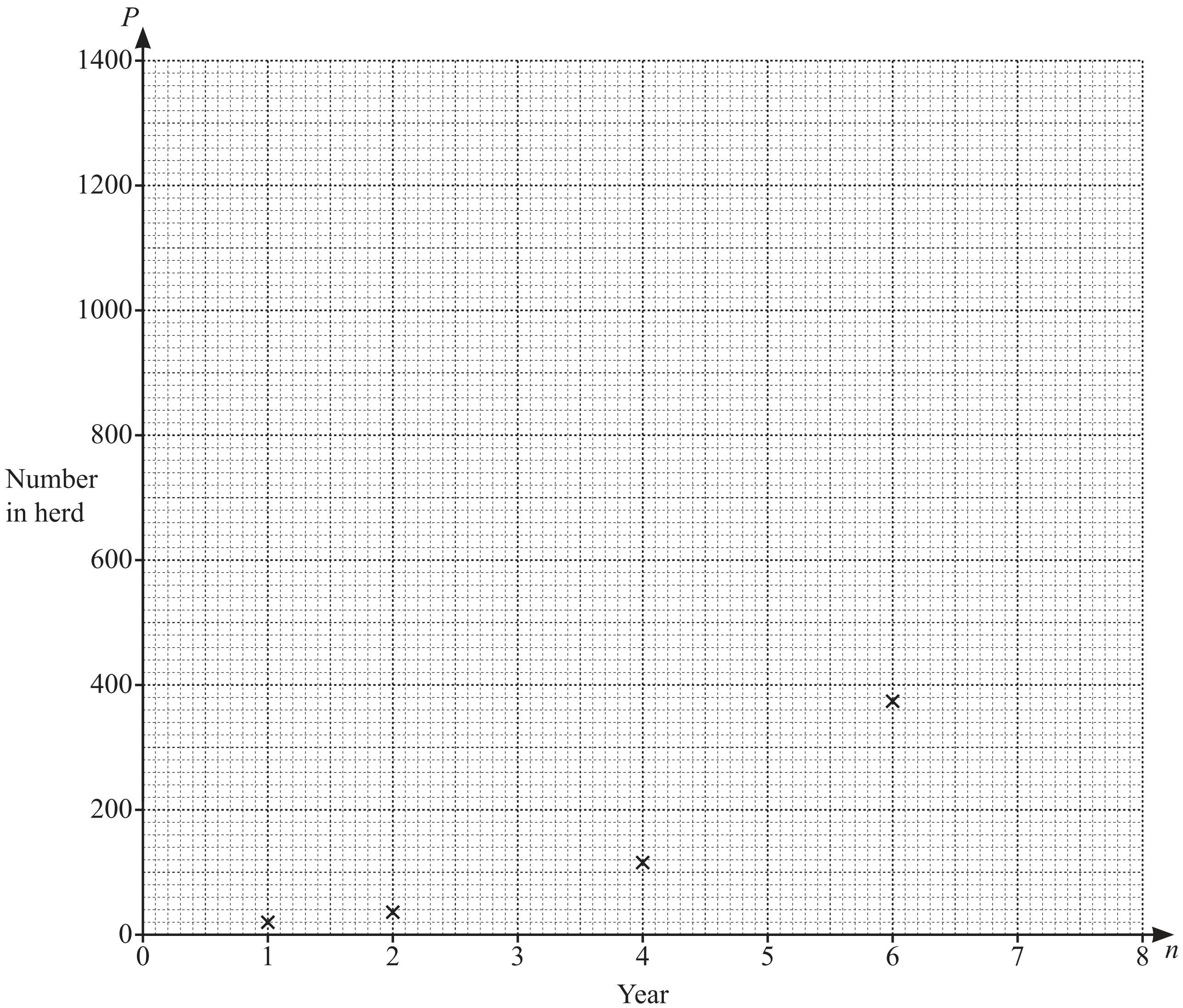
(b) **(i)** When the farmer finds P_{n+1} he rounds the value to the nearest **even integer**.
The table shows the number of deer in the herd, P_n, at the start of year n.

Use **part (a)(ii)** to complete the table.

Year (n)	1	2	3	4	5	6	7	8
Number in herd (P_n)	20	36		116		374		

[4]

(ii) Use your answers to **part (b)(i)** to plot the four missing points.



[2]

(c) The farmer models the data using $P = ab^{n-1}$.

Use the first two years in the table in **part (b)(i)** to find the value of a and the value of b and write down the model.

.... [3]

6 The farmer now models the data in the table on page 8 using $P = a(n-1)^2 + b(n-1) + c$.

(a) **(i)** Use the first year in the table in **Question 5(b)(i)** to show that $c = 20$.

[1]

(ii) Use year 2 and year 4 in the table in **Question 5(b)(i)** to write down a pair of simultaneous equations in terms of a and b.

....

.... [2]

(b) **(i)** Solve your simultaneous equations from **part (a)(ii)** and write down the model.

.... [4]

(ii) Sketch your model on the grid in **Question 5(b)(ii)** for $1 \leqslant n \leqslant 8$. [2]

(c) Is this a suitable model for the number of deer in the herd?
Give one reason for your answer.

....

.... [1]

7 The farmer now models the data in the table on page 8 using $P = a(n-1)^b$.

(a) Use the data for year 2 and year 4 in the table in **Question 5(b)(i)** to

(i) find a,

.... [2]

(ii) find b and write down the model.

.... [3]

(b) Give two reasons why the model in **part (a)** is not suitable for the number of deer in the herd.

1 ....

....

2 ....

.... [2]

This square dotty paper may be used for your diagrams in the investigation.